Exponential Functions Unit Review

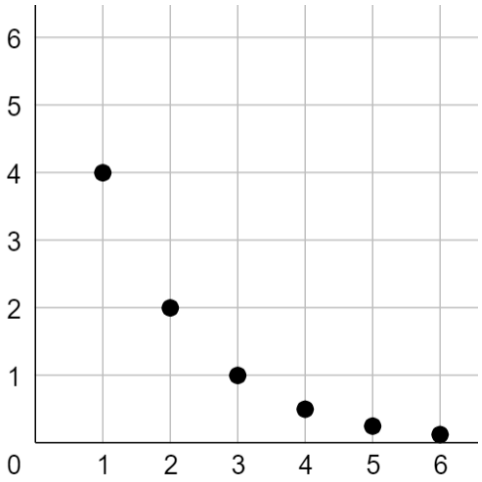
1. Find the thirtieth term in the arithmetic sequence that begins with terms $14, 20, 26, \ldots$

2. Find the sixteenth term in the geometric sequence with $g_7 = 30$ and $g_{10} = 15$
   (A) $\frac{15}{2}$
   (B) $\frac{15}{4}$
   (C) $\frac{15}{8}$
   (D) $\frac{15}{16}$

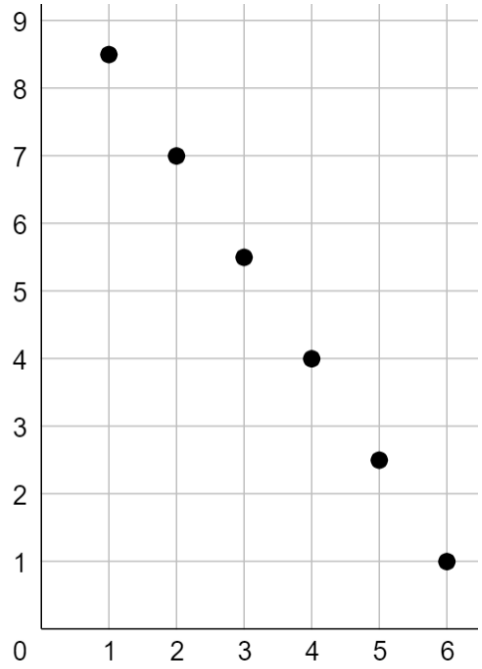
3. Find the common ratio of a geometric sequence with $g_4 = \frac{8}{5}$ and $g_7 = 200$.

4. Find the value of g_3 in a geometric sequence with $g_2 = \frac{4}{25}$ and $g_5 = \frac{5}{16}$.

5. Values of the geometric sequence g_n are graphed in the figure to the right. Determine a general formula for terms in this sequence.
   (A) $g_n = 4\left(\frac{1}{2}\right)^n$
   (B) $g_n = 8\left(\frac{1}{2}\right)^n$
   (C) $g_n = 4(2)^{n-1}$
   (D) $g_n = 8(2)^{n-1}$



6. Values of the arithmetic sequence a_n are graphed in the figure to the right. Determine a general formula for terms in this sequence.
   (A) $a_n = 8.5 - 1.5n$
   (B) $a_n = 8.5 - 3n$
   (C) $a_n = 10 - 1.5n$
   (D) $a_n = 10 - 3n$



7. (CALCULATOR) In the first week of a rumor spreading through a sufficiently large population, the number of people who hear the rumor each day can be modelled by a geometric sequence. On Tuesday 124 people hear the rumor. Two days later (Thursday), 279 people hear the rumor. How many people hear the rumor on the Wednesday that falls between those two days?

8. The table below gives values of a linear function $f(x)$.

x	-4	-2	3	5	8
$f(x)$	-39	?	3	15	33

Find $f(-2)$

9. Consider the graph of $f(x) = \frac{1}{3}(2)^{-x} + 4$.
   a. Is this function positive or negative over its domain?
   b. Which one of the following describes $f(x)$?
      (A) Increasing at a decreasing rate
      (B) Increasing at an increasing rate
      (C) Decreasing at a decreasing rate
      (D) Decreasing at an increasing rate
   c. Which one of the following describes the end behavior of $f(x)$?
      (A) As the input values of $f(x)$ decrease without bound, the output values increase without bound. As the input values of $f(x)$ increase without bound, the output values get arbitrarily close to 4.
      (B) As the input values of $f(x)$ decrease without bound, the output values decrease without bound. As the input values of $f(x)$ increase without bound, the output values get arbitrarily close to 4.
      (C) As the input values of $f(x)$ decrease without bound, the output values get arbitrarily close to 4. As the input values of $f(x)$ increase without bound, the output values decrease without bound.
      (D) As the input values of $f(x)$ decrease without bound, the output values get arbitrarily close to 4. As the input values of $f(x)$ increase without bound, the output values increase without bound.
10. Consider the graph of $g(x) = -4(e)^x$.
    a. Is this function concave up or concave down?
    b. Is this an increasing function or a decreasing function?
    c. Describe the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.
    d. Describe the end behavior of g as x decreases without bound. Express your answer using the mathematical notation of a limit.

11. $f(x)$ is an exponential function for which $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = 0$. Which one of the following describes $f(x)$?
    (A) $f(x)$ increases at a decreasing rate
    (B) $f(x)$ increases at an increasing rate
    (C) $f(x)$ decreases at a decreasing rate
    (D) $f(x)$ increases at an increasing rate

12. Bob is a window washer working on cleaning the windows of a large office building. At 2PM, he has washed 10% of the windows. At 4PM he has washed 70% of the windows. If Bob makes progress at a constant rate, at what time will he finish washing the windows?

13. (CALCULATOR) The value of Helen's new printer will depreciate with time. Though she paid $\$900$ for it today, it is expected that every year the value of the printer will decrease by 12%. Predict the value of the printer in 5 years. Round to the nearest cent.

14. The following table gives values of the function $f(x)$.

x	-4	-2	0	2	4	6	8
$f(x)$	10	7	4	1	-2	-5	-8

Which statement below best describes $f(x)$?

(A) $f(x)$ is modelled well by a linear function since the rate of change over consecutive equal-length input-value intervals is constant.

(B) $f(x)$ is modelled well by a linear function since the change in the average rate of change over consecutive equal-length input value intervals is constant.

(C) $f(x)$ is modelled well by a quadratic function since the rate of change over consecutive equal-length input-value intervals is constant.

(D) $f(x)$ is modelled well by a quadratic function since the change in the average rate of change over consecutive equal-length input value intervals is constant.

15. The following table gives values of the function $g(x)$.

x	-5	0	5	10	15	20	25
$g(x)$	27	17	10	6	5	7	12

Which statement below does NOT describes $g(x)$?

(A) $g(x)$ is modelled well by a quadratic function since the changes in output-value over consecutive equal-length input-value intervals form an arithmetic sequence.

(B) $g(x)$ is modelled well by a quadratic function since the change in the average rates of change over consecutive equal-length input value intervals is constant.

(C) $g(x)$ is modelled well by a quadratic function since the changes in output-value have a constant proportion of change over consecutive equal-length input-value intervals.

(D) $g(x)$ is modelled well by a quadratic function since the average rate of change over consecutive equal-length input-value intervals grows linearly.

16. The following table give values of the function $h(x)$ which is of the form $h(x) = a(b)^x + c$.

x	-6	-3	0	3	6	9
$h(x)$	18	2	-22	-58	?	-193

a. Find the value of $h(6)$

b. Is $a > 0$, or is $a < 0$?

c. Is $b > 1$, or is $0 < b < 1$?

17. The function $g(x) = 10(25)^{-\frac{1}{2}x-1}$ can be written in the form $g(x) = a(b)^x$. Find the value of a and b.

18. Let $f(x) = \frac{4}{8^x}$. Which of the following are equivalent representations of this function? Select all.

(A) $f(x) = \left(\frac{1}{2}\right)^x$

(B) $f(x) = \left(\frac{1}{2}\right)^{6x}$

(C) $f(x) = 4(2)^{-3x}$

(D) $f(x) = 8\left(\frac{1}{4}\right)^x$

(E) $f(x) = 2^{-3x+2}$

(F) $f(x) = \left(\frac{1}{2}\right)^{3x-2}$

19. The number of bacteria in a petri dish t days after Dr. Violet begins an experiment can be modelled by $P(t) = 1000(1.8)^t$. Which one of the following represents how many bacteria are in the petri dish w weeks after beginning the experiment?

(A) $1000(1.8)^{7w}$

(B) $1000(1.8)^{\frac{w}{7}}$

(C) $7000(1.8)^w$

(D) $7000(6.6)^w$

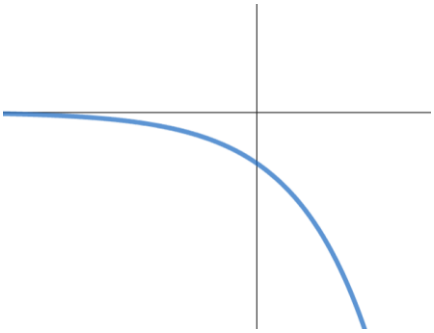
20. Every four months, the value of Dash's artwork increases by 5%. He sold a piece of art for $\$2000$ today.
    a. Find an equation that determines the value of that piece of art t years from today.
    b. Find an equation that determines the value of that piece of art t months from today.
    c. (CALCULATOR) Find the value of that piece of art will have in 3.5 years. Round to the nearest cent.

21. (CALCULATOR) Cadmium is a heavy metal used in pigments and batteries. When it gets into human bones, it has a half-life of 30 years. If Buddy has 900 mg of cadmium in his bones in January of 1990, predict how much cadmium he will have in his bones in January of 2040.

22. Let $f(x) = 4^x$. If the graph of $g(x)$ is the image of the graph of $f(x)$ after a horizontal dilation by a factor of $\frac{1}{2}$, which one of the following is $g(x)$?
    (A) $g(x) = 2^x$
    (B) $g(x) = 4^{x+2}$
    (C) $g(x) = 8^x$
    (D) $g(x) = 16^x$

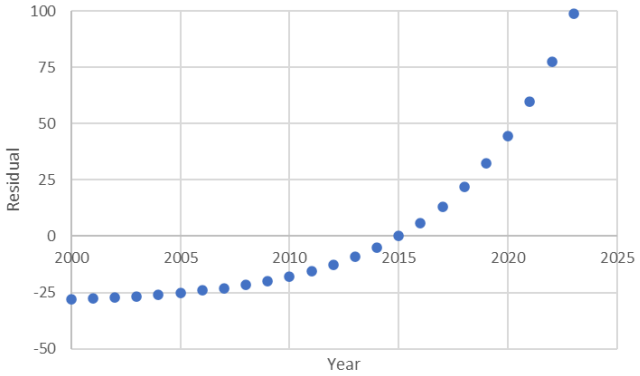
23. Let $h(x) = 4^x$. If the graph of $g(x)$ is the image of $h(x)$ after a vertical dilation by a factor of $\frac{1}{4}$, which one of the following is $g(x)$?
    (A) $g(x) = 4^{x-1}$
    (B) $g(x) = 4^{x+1}$
    (C) $g(x) = \left(\sqrt[4]{x}\right)^x$
    (D) $g(x) = 4^{4x}$

24. To the right is the graph of the exponential function $y = f(x)$. Which of the following transformations of $f(x)$ represents a positive function that exhibits exponential decay?
    (A) $g(x) = -f(-2x)$
    (B) $g(x) = -f(2x)$
    (C) $g(x) = 2f(-x)$
    (D) $g(x) = -2f(x)$



25. Let $f(x) = 9^x$. Let $g(x) = \left(\frac{1}{3}\right)^x$. Which one of the following statements accurately reflects the relationship between the graphs of these two functions.
    (A) The graph of $g(x)$ is the image of the graph of $f(x)$ after a reflection across the x-axis followed by a horizontal dilation by a factor of $\frac{1}{2}$.
    (B) The graph of $g(x)$ is the image of the graph of $f(x)$ after a reflection across the y-axis followed by a verical dilation by a factor of 2.
    (C) The graph of $g(x)$ is the image of the graph of $f(x)$ after a reflection across the y-axis followed by a horizontal dilation by a factor of 2.
    (D) The graph of $g(x)$ is the image of the graph of $f(x)$ after a reflection across the x-axis followed by a vertical dilation by a factor of $\frac{1}{2}$.

26. An exponential model is used to predict the grey wolf population in Yellowstone National Park over time. The figure shows a **plot of the residuals** of the exponential regression model. Which of the following statements about the exponential regression model is supported by this plot?



(A) The exponential model is appropriate as the pattern to the residuals seems to reflect a constant growth proportion.

(B) The exponential model is appropriate as the majority of the residuals are relatively close to zero.

(C) The exponential model is not appropriate because there is a clear pattern to the distribution of the residuals.

(D) The exponential model is not appropriate as the majority of the residuals are relatively close to zero.

27. (CALCULATOR) Consider the points included in the table below. Let $g(x)$ be an exponential regression model for these points.

x	-1	0	1	2	3	4	5
y	12.7	10.7	9.0	7.5	6.2	5.1	4.2

a. When $g(x)$ is expressed in the form $g(x) = a(b)^x$, what is the value of b? Round to three decimal places.

b. For every increase by one unit in x, by what percent does $g(x)$ decrease?

c. Find the residual value of the exponential model for $x = 2$?

28. Elephants require a lot of space. Jack is designing an app zookeepers may use to determine how many elephants can be kept at their facilities. Zookeepers input into the app the number of square feet available for elephants to roam, and the app calculates how many elephants can be kept in an area of that size. Sometimes the algorithm the app uses produces a non-integer number of elephants. What should Jack's app do in this circumstance?

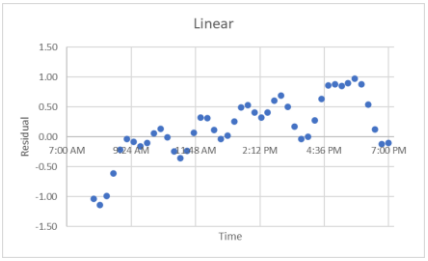
(A) Jack's app should round down to an integer. It is better to underestimate than overestimate in this context.

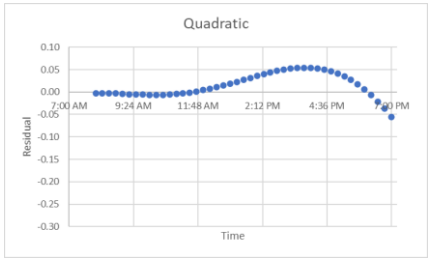
(B) Jack's app should round up to an integer. It is better to underestimate than overestimate in this context.

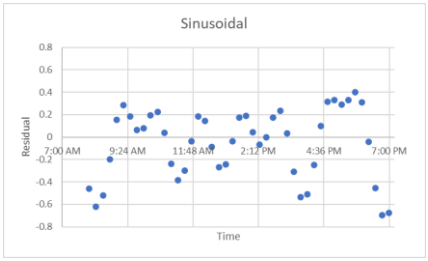
(C) Jack's app should round down to an integer. It is better to overestimate than underestimate in this context.

(D) Jack's app should round up to an integer. It is better to overestimate than underestimate in this context.

29. Edna is trying to model the rate at which people enter her store on Saturdays. She gathers data and computes regression models. The residual plot for the linear, quadratic, and sinusoidal model are shown below.







Based on these residual plots, which equation may best model the rate at which customer's enter Edna's store?

(A) $R(t) = 10.7t - 5.3$
(B) $R(t) = 4.1(1.3)^t$
(C) $R(t) = 20t(5 - t)$
(D) $R(t) = 5\sin(0.55t - 1.2) + 28.0$

30. Katie Ledecky is the most decorated American woman in Olympic history. In 2012, she was the youngest American to compete at the Olympics. Isa tracks how long it takes Katie to swim 800 meters on races on different days. She then constructs a regression model $S(t)$ that predicts Katie's 800 meter swim time (in seconds) t days after her Olympic debut. If $S(100) = 495.1$ and the residual associate with $t = 100$ is 4.3, then which of the following is true?

(A) The model S overestimates Katie Ledecky's 800 m swim time for $t = 100$. Her actual swim time is 499.4 seconds.
(B) The model S underestimates Katie Ledecky's 800 m swim time for $t = 100$. Her actual swim time is 499.4 seconds.
(C) The model S overestimates Katie Ledecky's 800 m swim time for $t = 100$. Her actual swim time is 490.8 seconds.
(D) The model S underestimates Katie Ledecky's 800 m swim time for $t = 100$. Her actual swim time is 490.8 seconds.

31. An 800 kg tree falls in the woods and begins to decompose. As it decomposes, the weight of the tree decreases. After t days, the weight of remaining tree is $W(t) = 800(0.997)^t$.

a. By what percent does the weight of undecomposed tree decrease each day?
b. The average rate of change between $t = 0$ and $t = 8$ is used to approximate the value of $W(10)$. Explain how the concavity of the graph of $W(t)$ explains whether the approximation based on average rate of change overestimates or underestimates the value of $W(10)$.

32. A hot pocket is warmed up in a microwave. 20 seconds after being removed from the microwave ($t = 20$), the hot pocket is $80°\text{C}$. Two minutes after being removed from the microwave ($t = 120$), the hot pocket is $40°\text{C}$.

a. Find the average rate of change of the temperature of the hot pocket from $t = 20$ to $t = 120$. Express your answer as a decimal approximation. Show the computations that lead to your answer.
b. Use the average rate of change found in part a to approximate the temperature of the hot pocket when $t = 140$. Show the work that leads to your answer.

Exponential Functions Unit Test Review Answers

1. 188
2. B
3. 5
4. $\frac{1}{5}$
5. B
6. C
7. 186
8. -27
9.
    a. Positive
    b. D
    c. A
10.
    a. Concave Down
    b. Decreasing
    c. $\lim_{x\to\infty} g(x) = -\infty$
    d. $\lim_{x\to-\infty} g(x) = 0$
11. A
12. 5 PM
13. $\$474.96$
14. A
15. C
16.
    a. -112
    b. $a < 0$
    c. $b > 1$
17. $g(x) = 10(25)^{-\frac{1}{2}x-1} = \frac{2}{5}\left(\frac{1}{5}\right)^{x}$. $a = \frac{2}{5}$ and $b = \frac{1}{5}$.
18. C, E, F
19. A
20.
    a. $P(t) = 2000(1.05)^{3t}$
    b. $P(t) = 2000(1.05)^{t/4}$
    c. $\$3338.24$
21. 283.482 mg
22. D
23. A
24. A
25. C
26. C
27.
    a. 0.831
    b. 16.9%
    c. 0.097
28. A
29. D
30. B

31.

a. The weight of undecomposed tree decreases by 0.3% each day

b. We are comparing the concave up function W to the outputs of the secant line passing through W at $t = 0$ and $t = 8$. Because W is concave up, the graph of the secant line is below the graph of W for $t > 8$. Thus, at $t = 10$, the secant line underestimates the actual value of $W(10)$.

32.

a. $\frac{40°\text{C}-80°\text{C}}{120\text{sec}-20\text{sec}} = -0.4\ \frac{°\text{C}}{\text{sec}}$ (you don't need units...including them is fine but if you write the wrong units, you don't get credit)

b. $\frac{T-40}{140-120} = -0.4$

$\frac{T-40}{20} = -0.4$

$T - 40 = -8$

$T = 32$

The hot pocket is approximately $32°\text{C}$ when $t = 140$